ISSN : 2319-2607 (Print); 2583-0856 (Online) IISUniv.J.Sc.Tech. Vol.10(1), 100-104 (2021)
RESEARCH ARTICLE

Joule Heating and Viscous Dissipation Effect on Heat Transfer and Unsteady Magnetohydrodynamic Flow over a Porous Stretching or Shrinking Sheet

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Abstract

The present article deals with the study of properties of Joule heating and viscous dissipation due to transfer of heat energy of magnetofluid over a porous stretching or shrinking sheet. The governing nonlinear coupled partial differential equations (PDE) are transformed into ordinary differential equations (ODE) and analyzed with shooting technique using Runge-Kutta fourth-order method. The relative nature of governing parameters such as sink or source prameter, time variable, permeability and magnetic parameter along with Eckert number and Prandtl number are calculated. The effect of these physical entity on velocity of fluid, temperature of fluid, Nusselt number and skin friction coefficient are obtained and discussed graphically using Matlab softawre.

Keywords: Heat transfer, Joule heating, MHD, porous medium, stretching/shrinking sheet.

AMS Mathematics Subject Classification (2010): 76W05

Introduction

The study of flows of boundary layer fluid flow near porous stretching sheet is of great importance in MHD and heat transfer. This type of flow not only occur in large number of engineering processes such as plastic sheets, food processing, polymer processing, annealing of copper wires and metallurgy but also in many natural phenomenon. The motion of stretching/shrinking sheet along with heating/cooling during such processes have a decisive influence on the quality of the intermediate and final products. Hence the assimilation of the heat transfer and flow uniqueness of the process has drawn the attention of many researchers. (Chen, 1997) discussed about magnetohydrodynamic (MHD) heat transfer over a non-isothermal stretching sheet. (Chen, 2010) studied the combined effect of Joule heating and viscous dissipation on MHD flow past a permeable stretching surface with the help of free convection and radiative heat transfer.. The flow past a stretching sheet discussed by (Crane, 1970). (Gupta and Gupta, 1977) considered heat and mass transfer on stretching sheet with suction and blowing. (Swain *et al.*, 2020) examine the influence of viscous dissipation as well as Joule heating on MHD and heat transfer past a stretching or shrinking sheet in porous medium. The effect of viscous dissipation and Joule heating of the magnetohydrodynamic fluid flow over the stretching sheet has also been studied by (Sharma and Sinha, 2017). (Siddheshwar *et al.*, 2013) discussed the behaviors of flow and heat transfer in a Newtonian fluid with temperature dependent properties. (Reddy *et al.*, 2015) examine the Effect of viscous dissipation and heat source/sink on unsteady magnetohydrodynamic motion on stretching sheet. (Wang, 1989) investigated free convection on a vertical stretching surface. Aim of the current work is to study the combined effect of Joule heating and viscous dissipation over a Porous stretching or shrinking sheet.

Governing equations

Consider the motion of an incompressible Newtonian fluid over a stretching sheet in the existence of viscous dissipation and Joule heating. Let fluid is electrically conducting and is moving on the sheet which is placed horizontally along x-direction and let y-direction is taken vertical to horizontal axis. Let the magnetic force is acting perpendicular to the sheet, governing continuity equation, energy equation and Navier Stoke's equations after reduction can be given as

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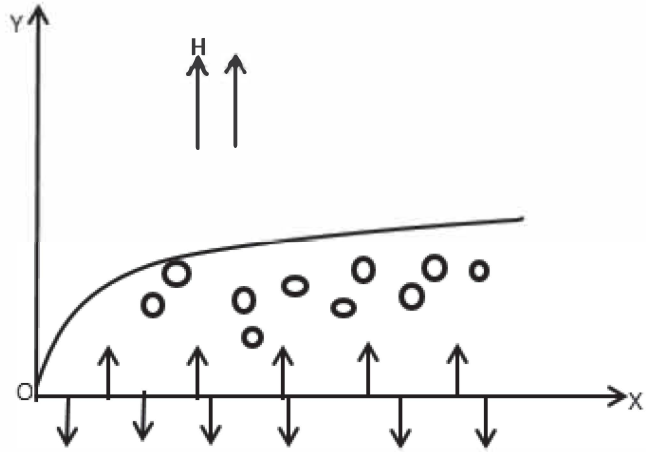

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H^2}{\rho}u-\frac{\upsilon}{K'}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q}{\partial y}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho C_p}(T-T_\infty)+\frac{\sigma H^2}{\rho C_p}u^2 \tag{3}$$

The corresponding conditions at boundaries are

At $y=0$: $u=U_w(x,t),\ v=0,\ T=T_{wall}(x,t)$

$As\ y\rightarrow\infty$: $u=0$ and $T=T_\infty$ (4)

Where u, v are the velocity components in x-axis and y-axis respectively, ρ is the density, T is the temperature, υ is kinematic viscosity, μ is the coefficient of viscosity, Cp is specific heat at constant pressure, σ is the electrical conductivity, a is the thermal diffusivity, Q is the volumetric heat absorption rate, q is the radiative heat flow and H is the magnetic field applied normal to the surface. Let the stretching velocity, surface temperature, variable magnetic field and heat generation/absorption level be defined as follows

$$U_w(x,t)=m\frac{x}{t}\quad T_w(x,t)=T_\infty+n\frac{x}{t},\quad H=\frac{H_0}{\sqrt{t}},\quad Q=\frac{Q_0}{t} \tag{5}$$

where $H0$, $Q0$, m, n are some constants. The flux of radiative heat, according to Roseland diffusion approximation is given by :

$$q=\frac{4\sigma'}{3\kappa'}\frac{\partial T^4}{\partial y},$$

where k' and σ' are Rosseland mean coefficient of absorption and Stefan-Boltzman constant respectively.

Since the difference in temperature inside the boundary layer is much low therefore by using the Taylor series expansion $T4$can be expressed as the linear power of T.

$$T^4\cong 4T_\infty^3T-3T_\infty^4$$

and thus radiative heat flux gradient becomes

$$\frac{\partial q}{\partial y}=-\frac{16\sigma' T_\infty^3}{3\kappa'}\frac{\partial^2 T}{\partial y^2}$$

Hence the energy equation reduced to

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+a(1+L)\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}(T-T_\infty)+\frac{\sigma H^2}{\rho C_p}u^2$$

where $L=16\sigma' T_\infty^3/3k'k$ is the radiation parameter. k is thermal conductivity .

Introducing non-dimensional parameters,

$$\eta=y\sqrt{\frac{U_w}{\upsilon x}},\qquad \phi=\sqrt{U_w\upsilon x}\,f(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

where $\varphi(x,y)$ is stream function, with $u=\frac{\partial\varphi}{\partial y},\ v=-\frac{\partial\varphi}{\partial x}$ satisfying the continuity equation identically.

Equation of motion reduced to,

$$\frac{m^{3/2}xy}{2(t)^{5/2}\sqrt{\upsilon}}f''+\frac{mx}{t^2}f'+\frac{m^2x}{t^2}(f')^2-\frac{m^2x}{t^2}ff''=\frac{m^2x}{t^2}f'''-\frac{\sigma H^2mx}{\rho t}f'-\frac{\upsilon mx}{Kt}f'$$

$$\Rightarrow f'''+ff''-(f')^2-Af'-\frac{1}{m}\left[f'+\frac{\eta}{2}f''\right]-\frac{1}{K'}f'=0$$

Similarly, the equation of energy,

$$nx\theta+\frac{nx}{2}y\sqrt{\frac{m}{\upsilon t}}\theta'+mnxf'\theta-mnxf\theta'=a(1+L)\frac{mnx}{\upsilon}\theta''$$

$$+\frac{\upsilon}{\rho C_p}\frac{x^2m^3}{\upsilon t}f''^2+\frac{Q_0nx}{\rho C_p}\theta+\frac{\sigma H_0m^2x^2}{\rho C_pt^3}f'^2$$

$$(1+L)\theta''+\Pr(f\theta'-f'\theta)-\frac{\Pr}{m}\left(\theta+\frac{\eta}{2}\theta'\right)$$

$$+\Pr\left[Ec\left((f'')^2+A(f')^2\right)+S\theta\right]=0$$

Thus, the equations of motion and energy are

$$f'''+ff''-(f')^2-Af'-\frac{1}{m}\left[f'+\frac{\eta}{2}f''\right]-\frac{1}{K'}f'=0 \tag{6}$$

$$(1+L)\theta''+\Pr(f\theta'-f'\theta)-\frac{\Pr}{m}\left(\theta+\frac{\eta}{2}\theta'\right)$$

$$+\Pr\left[Ec((f'')^2+A(f')^2+S\theta\right]=0 \tag{7}$$

where, $A=\frac{\sigma H_0^2}{\rho m}$, $K'=\frac{mK}{\upsilon t}$, $Ec=\frac{U_w^2}{C_p(T_w-T_\infty)}$, $Pr=\frac{\upsilon}{a}$,

$S=\frac{Q_0}{\rho C_p m}$, Thus, the equations of motion and energy are are magnetic parameter, permeability parameter, Eckert number, Prandtl number and sink parameter respectively.

The boundary condition in term of f and θ

$$f'(\eta)=1,\ f(\eta)=0,\ \theta(\eta)=1 \text{ at } \eta=0$$

$$f'(\eta)=0,\ \theta(\eta)=0 \text{ at } \eta=\infty \tag{8}$$

The skin-friction coefficient and Nusselt number at the plate surface are given by

$$C_f=\frac{2\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho U_w^2},\quad Nu_l=\frac{-xK\left(\frac{\partial T}{\partial y}\right)_{y=0}}{K(T_w-T_\infty)},$$

$$\frac{1}{2}C_f\sqrt{Re_l}=f''(0),\ Nu_l/\sqrt{Re_l}=-\theta'(0), \tag{9}$$

where Re_l is local Reynolds number.

Method of Solution

The above equations (6) and (7) along with the boundary conditions (8) being highly nonlinear in nature are solved by using Runge-Kutta-fourth order method with shooting technique using mathematical tool (Matlab Software). To use of above techniques first we convert the second order differential equations (6) and (7) to a set of first order diffrential equations.

The reduced equations are

$$g_1'=g_2,\quad g_2'=g_3,\quad g_3'=-g_1g_3+g_2^2+Ag_2+\frac{1}{m}\left(g_2+\frac{\eta}{2}g_3\right)+\frac{1}{K'}gs_2,$$

$$g_4'=g_5,\quad g_5'=\frac{Pr}{(1+L)}\left((g_1g_5-g_2g_4)-\frac{1}{m}\left(g_4+\frac{\eta}{2}g_5\right)+\left(Ec\left(g_3^2+Ag_2^2\right)+Sg_4\right)\right)$$

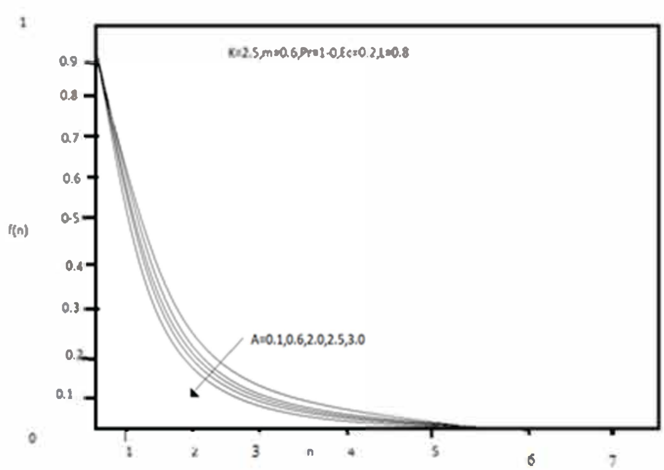

Fig.1. Graph between velocity and η (for different values of magnetic parameter)

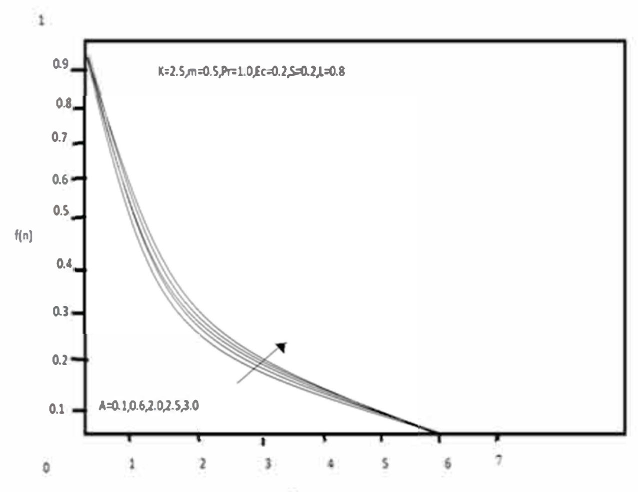

Fig.2. Graph between temperature and η (for different values of magnetic effect parameter.)

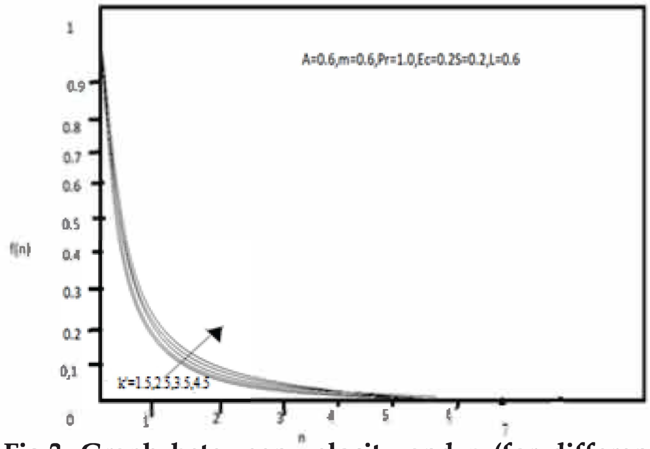

Fig.3. Graph betewen velocity and η (for different values of Permeability Parameter.)

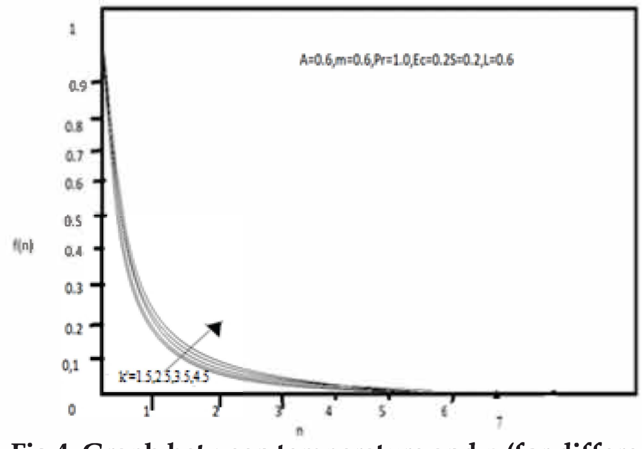

Fig.4. Graph between temperature and η (for different values of Permeability parameter.)

The corresponding boundary condition are reduced to

$$g_1(0)=0, \quad g_2(0)=1, \quad g_4(0)=1, \quad g_2(\infty)=0, \quad g_4(\infty)=0.$$

In order to solve above equations, we need g_1, g_2, g_3, g_4 and g_5 at $\eta=0$, which has been calculated with the help of MATLAB software.

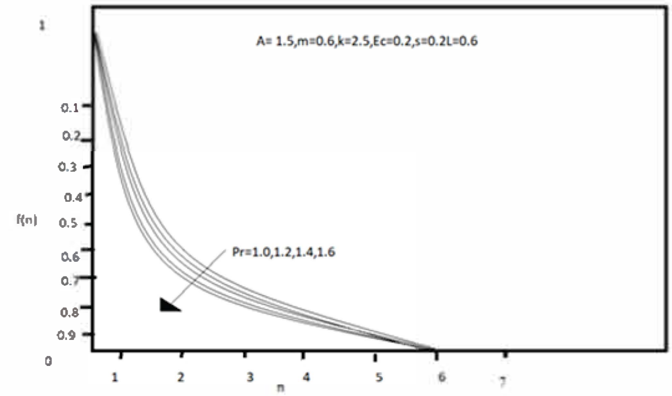

Fig. 5. Graph between temperature and η (for different values of Prandtl number.)

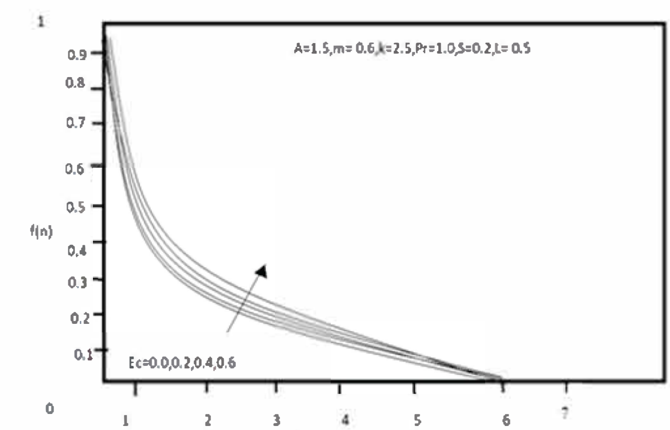

Fig.6. Graph between temperature and η (for different values Eckert number.)

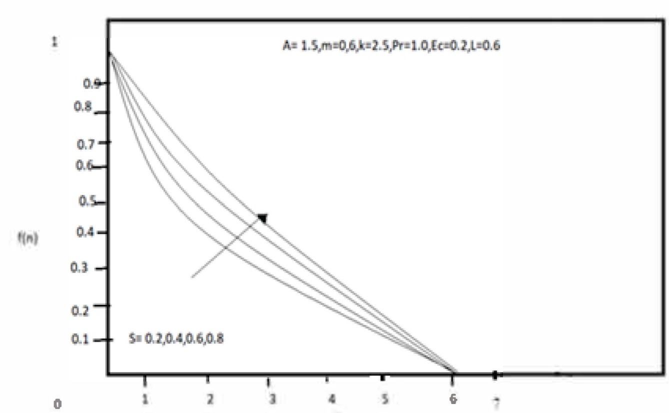

Fig.7. Graph between temperature and η (for different values Heat source.)

Results and Discussion

The effects of various physical entity like permeability, magnetic parameter, Nusselt number and Prandtl etc. are explained elaborately. It is observed from Fig.1 and 2 Fig. 3 and 4 represent that as permeability parameter K increases then fluid velocity increases while temperature decreases. This result enhanced the value of skin friction and Nusselt number .

It has been seen from Fig. 5 that as Prandtl number increases velocity of the fluid decreases.

As noted from Fig. 6 and 7 fluid temperature increase with increase in heat sink or source parameter and Eckert number Ec.

Conclusion

The effect of viscous dissipation on fluid flow and Joule heating is discussed elaborately. Some nice results are also given below.

- When magnetic parameter increases, temperature increases while velocity decreases.
- There is raise in temperature when Eckert number increase while it falls with increase in Prandtl number.
- Permeability parameter increase by increase of fluid velocity while it decreases with temperature parameter.
- Temperature is significantly improved when there is improve in heat source/sink parameter.

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